

CONTROL ÁLGEBRA

Octubre 2005

1.- Calcula y simplifica:

(2 puntos)

a) $x^2(x+3) - (x^2 - 2x + 3)(x^2 - 1) =$

b) $(x+2)(x-2)x - (x-2)^2 - x^3 + 4 =$

2.- Dados los polinomios:

(2 puntos)

$$P(x) = x^4 - 2x^3 + 2x - 1$$

$$Q(x) = 2x^4 + 3x^3 - 2x^2 - 3x$$

Descomponlos en factores y halla su m.c.m. y su M.C.D.

2.- Efectúa las siguientes operaciones y simplifica:

(6 puntos)

a) $\frac{x}{x-2} - \frac{x}{x-1} - \frac{x}{x^2 - 3x + 2} =$

b) $\frac{x-1}{3x^2} \cdot \frac{x^2 - x}{x^2 - 1} =$

c) $\frac{x^2 - 3x}{x+2} : \frac{x^2 - 5x + 6}{x^2 - 4} =$

d) $\left(\frac{1}{1+x} + \frac{2x}{1-x^2} \right) \cdot \frac{1-x}{x} =$

SOLUCIONES

$$1.- a) x^2(x+3) - (x^2 - 2x + 3)(x^2 - 1) = x^3 + 3x^2 - x^4 + x^2 + 2x^3 - 2x - 3x^2 + 3 = -x^4 + 3x^3 + x^2 - 2x + 3$$

$$b) (x+2)(x-2)x - (x-2)^2 - x^3 + 4 = (x^2 - 4)x - (x^2 - 4x + 4) - x^3 + 4 = x^3 - 4x - x^2 + 4x - 4 - x^3 + 4 = -x^2$$

$$2.- P(x) = x^4 - 2x^3 + 2x - 1$$

$$\text{Div}(1)=1, -1$$

$$\begin{array}{r|rrrrr} & 1 & -2 & 0 & 2 & -1 \\ 1 & & 1 & -1 & -1 & 1 \\ \hline & 1 & -1 & -1 & 1 & 0 \\ 1 & & 1 & 0 & -1 & \\ \hline & 1 & 0 & -1 & 0 & \end{array}$$

$$x^2 - 1 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

$$P(x) = (x-1)(x-1)(x-1)(x+1)$$

$$Q(x) = 2x^4 + 3x^3 - 2x^2 - 3x = x(2x^3 + 3x^2 - 2x - 3) \quad \text{Div}(3)=1, -1, 3, -3$$

$$\begin{array}{r|rrrr} & 2 & 3 & -2 & -3 \\ 1 & & 2 & 5 & 3 \\ \hline & 2 & 5 & 3 & 0 \end{array}$$

$$2x^2 + 5x + 3 = 0$$

$$x = \frac{-5 \pm \sqrt{25 - 4 \cdot 2 \cdot 3}}{4} = \frac{-5 \pm 1}{4} = \begin{cases} -1 \\ -\frac{6}{4} = -\frac{3}{2} \end{cases}$$

$$\text{Luego } Q(x) = 2x(x-1)(x+1) \left(x + \frac{3}{2} \right)$$

$$P(x) = (x-1)^3(x+1)$$

$$\text{m.c.m.} = x(x-1)^3(x+1)(2x+3)$$

$$Q(x) = x(x-1)(x+1)(2x+3)$$

$$\text{M.C.D.} = (x-1)(x+1)$$

3.-

$$a) \frac{x}{x-2} - \frac{x}{x-1} - \frac{x}{x^2-3x+2} = \frac{x(x-1)}{(x-1)(x-2)} - \frac{x(x-2)}{(x-1)(x-2)} - \frac{x}{(x-1)(x-2)} =$$

$$x^2 - 3x + 2 = (x-1)(x-2) \Rightarrow \text{m.c.m.} = (x-1)(x-2)$$

$$= \frac{x^2 - x - x^2 + 2x - x}{(x-1)(x-2)} = \frac{-2x + 2x}{(x-1)(x-2)} = \frac{0}{(x-1)(x-2)} = 0$$

$$b) \frac{x-1}{3x^2} \cdot \frac{x^2-x}{x^2-1} = \frac{(x-1)(x^2-x)}{3x^2(x^2-1)} = \frac{(x-1)x(x-1)}{3x^2(x+1)(x-1)} = \frac{x-1}{3x(x+1)}$$

$$c) \frac{x^2 - 3x}{x + 2} : \frac{x^2 - 5x + 6}{x^2 - 4} = \frac{(x^2 - 3x)(x^2 - 4)}{(x + 2)(x^2 - 5x + 6)} = \frac{x(x - 3)(x - 2)(x + 2)}{(x + 2)(x - 3)(x - 2)} = x$$

$$x^2 - 3x = x(x - 3) \quad x^2 - 5x + 6 = 0 \Rightarrow x = \frac{5 \pm \sqrt{25 - 24}}{2} = \begin{cases} 3 \\ 2 \end{cases}$$

$$x^2 - 4 = (x + 2)(x - 2) \quad x^2 - 5x + 6 = (x - 3)(x - 2)$$

$$d) \left(\frac{1}{1+x} + \frac{2x}{1-x^2} \right) \cdot \frac{1-x}{x} = \left(\frac{1-x}{(1-x)(1+x)} + \frac{2x}{(1-x)(1+x)} \right) \cdot \frac{1-x}{x} =$$

$$\left. \begin{array}{l} 1+x = (1+x) \\ 1-x^2 = (1+x)(1-x) \end{array} \right\} m.c.m. = (1+x)(1-x)$$

$$\left(\frac{1-x+2x}{(1-x)(1+x)} \right) \cdot \frac{1-x}{x} = \frac{(1+x)(1-x)}{(1-x)(1+x)x} = \frac{1}{x}$$